Class Notes

• 15 min “The Lives of Stars” reading + quiz
• Reminder: math quiz Friday
  – Review problem sets #1, 2, and in-class notes
• And other things I’ve already harrassed you about.
Questions from Last Time

• How can we tell how large the Universe is when light & time are not always measurable?
• Is force necessary for expansion? What are causes & implications of (1) expansion, (2) acceleration, and (3) acceleration of acceleration?
• In what material does light go slower than $3 \times 10^5$ km/s?
• How was CMBR discovered?
• How does c determine size of Universe?
• What would happen to rope?
• General question of applying all the different laws.
• How did matter win over anti-matter right after Big Bang?
• What is redshift?
• Is the Universe going to collapse? How do we know?
Questions from Last Time

- How can we tell how large the Universe is when light & time are not always measurable? That is observable Universe.
- Is force necessary for expansion? What are causes & implications of (1) expansion, (2) acceleration, and (3) acceleration of acceleration? Big Bang and dark energy $\Lambda$.
- In what material does light go slower than $3 \times 10^5$ km/s? Everything but a vacuum.
- How was CMBR discovered? By accident.
- How does $c$ determine size of Universe? All we can observe.
- What would happen to rope? Many things.
- General question of applying all the different laws. Office hours.
- How did matter win over anti-matter right after Big Bang? Statistical imbalance. (Also good presentation topic; see CP violation.)
- What is redshift? $z = \Delta/\lambda$.
- Is the Universe going to collapse? How do we know? No, We’ll discuss more later.
Objects in the Universe: Stars (Part I)

Kathy Cooksey
AY5 Introductory Astronomy
Wednesday, July 9, 2008
Stars

- **Mass** determines majority of characteristics
  - Luminosity, size, lifetime...
- Life is about balance (**equilibrium**)
- Primarily composed of H, He
  - Some heavier elements (**metals**): C, N, O, Cr, Fe...
- Fuse H into He for majority of life
  - **Main sequence**

Focus of this lecture.
What is the closest star to Earth?
Our Sun

- 8.3 light-min. from Earth
- \( L_{\text{SUN}} = 3.8 \times 10^{26} \text{ W} \)
  - \( m = -26.7 \text{ mag} \)
- \( M_{\text{SUN}} = 1.99 \times 10^{30} \text{ kg} \)
  - \( 333,000 M_{\text{EARTH}} \)
- \( R_{\text{SUN}} = 695,000 \text{ km} \)
  - \( 109 R_{\text{EARTH}} \)
- \( T_{\text{surf}} = 5800 \text{ K} \)
- Spectral type: G2V
- \( t_{\text{age}} = 4.6 \text{ Gyr} \)
Solar Structure

Visible surface

Corona
$10^6$ K

Photosphere
5800 K

Radiative Zone

Core

Convective Zone

Chromosphere
$10^4$ K

Visible during solar eclipse.

Fusion

Largest UV source

(stloe.most.go.th)
Solar Interior

- Not directly observable
  - Except by neutrinos
- Radiative diffusion
  - Photons random walk through high density interior (scattering).
  - 170,000 yr to escape!
- Convection
  - Another energy transport.
- Models based on observations

(Solar Astrophysics)
Solar Neutrinos

- Direct probe of solar interior
  - Since neutrinos rarely interact
  - Observed vs predicted rate
- Also observe neutrinos from other astronomical sources

Super-Kamiokande (Cosmic Perspective)
Solar Mass

• Measured from planetary motion
  – Newton’s generalization of Kepler’s third law
  – $M_{\text{EARTH}}$ is negligible ($<< M_{\text{SUN}}$)

\[ p^3 = \frac{4\pi^2}{G(M_{\text{SUN}} + M_{\text{EARTH}})} a^3 \]

Period of orbit \hspace{2cm} Average distance
Solar Composition

73.4% H, 25% He, 0.2% C, 0.09% N, 0.8% O, (< 0.5% Ne, Mg, Si, S, Fe)
Solar Structure Models

• Input: observables
  – Physics (gravity, nuclear fusion, etc)
  – Composition (H, He, metals)
  – Mass

• Output: temperature, pressure, density profiles (i.e., vs depth)
  – Predict: $R_{\text{SUN}}$, $T_{\text{surf}}$, $L_{\text{SUN}}$, $t_{\text{age}}$, ...
  – Verify: helioseismology
Let’s talk about these properties

L, T,...
Luminosity and Flux

- Luminosity is energy radiated per second
  - J/s
  - Cooling process
- Flux is luminosity per unit area
  - J/s/m²
  - Inverse-square Law

\[ F = \frac{L}{4\pi r^2} = \sigma T^4 \]

Stefan-Boltzmann Law (Cosmic Perspective)
Magnitude

- Smaller magnitude, brighter object
  - Humans see apparent magnitudes 1 to 6.
- Each magnitude is factor of 2.5 in brightness.
  - Formally, 1st magnitude star is 100 times brighter than 6th magnitude star.
- **Absolute magnitude** defined at distance of 10 pc.

Magnitude of two objects:

\[ m_1 - m_2 = -2.5 \log_{10} \left( \frac{F_1}{F_2} \right) \]

Apparent and absolute magnitude of same object:

\[ m - M = -5 + 5 \log_{10} d \]
Relating to Hubble’s Law

- **Standard candles** are objects with known **absolute magnitude** $M$.
  - Observe **apparent magnitude** $m$.
  - Measure distance to object $d$.
  - Observe redshift $z$.
  - Measure Hubble constant $H_0$ ($cz = H_0d$).

- **Process** to create **cosmological distance ladder**.

\[ m - M = -5 + 5 \log_{10} d \]
Solar Luminosity (Recap)

- $L_{\text{SUN}} = 3.8 \times 10^{26} \text{ W} = 3.8 \times 10^{26} \text{ J/s}$
  - Measured and predicted by models.
  - $F_{\text{surf}} = 6.3 \times 10^7 \text{ J/s/m}^2 = \sigma T_{\text{surf}}^4$
    - $T_{\text{surf}} = 5800 \text{ K}$
- $m_{\text{SUN}} = -26.74 \text{ mag (appears bright)}$
  - Observed because $d = 4.8 \times 10^{-6} \text{ pc}$
  - Observed in V(isible) band.
- $M_{\text{SUN}} = 4.83 \text{ mag (not absolutely bright)}$
  - Defined at $d = 10 \text{ pc}$
Why does the Sun shine?

I’m attacking a misconception.
Why does the Sun shine?

Force of gravity compresses gas.
- Pressure, density, *temperature* rise.
- Radiation (eventually) escapes.
- Sun shines.
Hydrostatic Equilibrium

\[ F_{\text{grav}} = \frac{GMm}{r^2} \]

\[ F_{\text{out}} = ? \]
Outward Pressure

• Pressure is force per unit area
  – Balances $F_{\text{grav}}$

• Due to gas pressure or radiation pressure
  – $P_{\text{gas}} = nkT$
    • ($n = \text{particle \# density}$)
  – $P_{\text{rad}} = F/c$
    • ($F = \text{flux}$)

\[
P_{\text{out}} A = F_{\text{grav}} = \frac{GMm}{r^2}
\]
How does the Sun continue to shine?

For 4.6 Gyr…
Thermal Equilibrium

• Sun loses energy as it shines.
  – Luminosity is energy lost per second.
• Energy loss decreases pressure, density, temperature.
  – Less resistance to gravity.
• UNLESS energy is replaced…
  – to increase pressure, density, temperature.
Nuclear Fusion

• Source of energy to maintain equilibria
  – And keep shining
• Primary reaction in Sun: proton-proton chain
  – 4 protons ($^1\text{H}$) make one $^4\text{He}$ and 26.7 MeV of energy

(Note: note same process as He production in early Universe.)
Conditions for Fusion

- \( T_{\text{core}} = 15 \times 10^6 \) K
  - \( <E_{\text{particle}}> = 1.3 \text{ keV} \)
  - \( \lambda_{\text{peak}} = 1 \text{ nm (UV)} \)
- \( P_{\text{core}} = 3.5 \times 10^{11} \) atm
  - Due to mass “above”
- \( \rho_{\text{core}} = 1.5 \times 10^5 \) kg/m\(^3\)
  - \( <\rho_{\text{SUN}}> = 1409 \) kg/m\(^3\)
  - \( \rho_{\text{H}_2\text{O}} = 1000 \) kg/m\(^3\)

\[
P_{\text{core}} = n k T_{\text{core}}
\]
Stability of Hydrostatic Equilibrium

- Nuclear fusion rate very sensitive to $T_{\text{core}}$.
- Gravity and radiation pressure work to stay in equilibrium.
Stars

Beyond our Sun (still main sequence)
Sun and Stars

Sun is not special*

- \( M = 1 \, M_{\text{SUN}} \)
- \( L = 1 \, L_{\text{SUN}} \)
- \( T_{\text{SURF}} = 5800 \, \text{K} \)
- \( R = 1 \, R_{\text{SUN}} \)
- Proton-proton chain
  - Internal structure
- \( Z = 1 \, Z_{\text{SUN}} \)
- \( t_{\text{age}} = 4.6 \times 10^9 \, \text{yr} \)
- Single*

- \( 0.08 \text{–} 100 \, M_{\text{SUN}} \)
- \( 10^{-4} \text{–} 10^6 \, L_{\text{SUN}} \)
- \( 3000 \text{–} 40,000 \, \text{K} \)
- \( 0.05 \text{–} 15 \, R_{\text{SUN}} \)
- p-p chain, CNO cycle
  - Depends on reaction
- \( 10^{-5} \text{–} 2 \, Z_{\text{SUN}} \)
- \( 2 \times 10^6 \text{–} 10^{12} \, \text{yr} \)
- Single, binary, multiple
Stellar Mass: “I’m the decider”
M-R Relation

\[ \frac{R}{R_{\text{SUN}}} = \left( \frac{M}{M_{\text{SUN}}} \right)^{0.8} \]

- Nuclear fusion very sensitive to \( T_{\text{core}} \).
  - \( T_{\text{core}} \) does not vary much between stars.
- Radius adjusts to compensate for mass.
  - Hydrostatic equilibrium

(daviddarling.info)
Stellar Temperature

- Observe $T_{\text{surf}}$ (Wien’s Law: $\lambda_{\text{peak}} = \kappa T^2$)
  - Physics says $T_{\text{surf}} \propto T_{\text{core}}$.
- Hydrostatic equilibrium sets $T_{\text{core}} \propto M/R$.
  - But fusion very sensitive to $T_{\text{core}}$.
  - Radius adjusts and $M \propto R$.
- $T_{\text{surf}}$ almost constant over mass.
Stellar Spectral Classification

‘Oh Boy An F Grade Kills Me’

Hot

Cool

(U WA)

(Colombia)

Less Lines

More Lines

Logarithmic brightness

Angstroms

(3000 4000 5000 6000 7000 8000)
Stellar Luminosity Classes

- V: main sequence stars
  - Dwarfs
- IV: subgiants
- III: giants
- II: bright giants
- I: supergiants
Stellar Structure

Convection

Radiative Diffusion

$T_{\text{core}} \approx 40 \times 10^6 \text{ K}$

$T_{\text{core}} \approx 15 \times 10^6 \text{ K}$

$T_{\text{core}} \approx 4 \times 10^6 \text{ K}$

$T_{\text{core}}$ mostly insensitive to mass

Note: $L$ function of $T_{\text{suf}}$ and $A_{\text{surf}}$. (

(bramboroson.com)
M-L Relation

• Empirical (observed)

\[ L \propto M^\alpha \]

\[ \alpha \approx 3-4 \]

• Changes for lowest- and highest-mass stars

\[ \frac{L}{L_{SUN}} = \left( \frac{M}{M_{SUN}} \right)^4 \]

(Pogge at OSU)

(NAU)
Stellar Lifetime

- Lifetime increases as mass decreases
- More mass → More $F_{\text{grav}}$
  → More $P_{\text{core}}$
  → Higher $T_{\text{core}}$
  → More fusion
  → Higher $L$ (consumption)
- Think SUV vs SmartCar

\[ t_{\text{age}} \propto \frac{M}{L} \approx \frac{M}{M^{3.5}} \approx M^{-2.5} \]
Stellar Fusion

- **Proton-proton chain** dominates at lower $T_{\text{core}}$
- **CNO cycle** dominates at higher $T_{\text{core}}$
- Other fusion discussed later (e.g., $3 \text{He} \rightarrow \text{C}$)

(Wikipedia)

Main sequence
($H \rightarrow \text{He}$)
processes

Relative energy production rate

(Wikipedia)
Stars

- **Mass** determines majority of characteristics
  - Luminosity, size, lifetime…
- Life is about balance *(equilibrium)*
- Primarily composed of H, He
  - Some heavier elements *(metals)*: C, N, O, Cr, Fe…
- Fuse H into He for majority of life
  - Main sequence

Focus of this lecture.
Let’s synthesize.

\[ M, F_{\text{grav}}, P_{\text{out}} = P_{\text{core}}, T_{\text{core}}, T_{\text{surf}}, L, F \]
Main Sequence Equilibria 1

Gravity (mass) \[\rightarrow\] Hydrostatic Eq. \[\rightarrow\] Pressure (out)
Main Sequence Equilibria 2

Gravity (mass) \rightarrow Hydrostatic Eq. \rightarrow Pressure (out)

Luminosity \rightarrow Thermal Eq. \rightarrow Energy source
Main Sequence Equilibria 4

Gravity (mass) ➔ Hydrostatic Eq. ➔ Pressure (out) ➔ T_{core} ➔ Luminosity ➔ Thermal Eq. ➔ Energy source ➔ Gravitational contraction

Because MAIN SEQUENCE ➔ Nuclear fusion
Main Sequence Equilibria 5

Gravity (mass)  \xrightarrow{\text{Hydrostatic Eq.}}  \text{Pressure (out)}

Gravitational contraction

Stefan-Boltzmann Law

\[ T_{\text{surf}} \propto \frac{L}{R^2} \]

Luminosity

Thermal Eq.

Nuclear fusion

Energy source

Gravitational contraction
Main Sequence Equilibria 6

Gravity (mass) \(\rightarrow\) Hydrostatic Eq. \(\rightarrow\) Pressure (out)

Luminosity \(\rightarrow\) Thermal Eq. \(\rightarrow\) Nuclear fusion

Gravitational contraction

Core is input
Surface is output

\[ T_{\text{surf}} \propto \frac{L}{R^2} \]

\[ T_{\text{core}} \]
Main Sequence Equilibria 7

Gravity (mass)

Luminosity

L \propto M^\alpha 
(empirical)

Hydrostatic Eq.

\[ T_{\text{surf}} \propto \frac{L}{R^2} \]

Pressure (out)

Gravitational contraction

Energy source

Nuclear fusion

M-L relation folds in rest of diagram.

T_{\text{core}}

Thermal Eq.

\[ T_{\text{surf}} \propto \frac{L}{R^2} \]
Nuclear fusion very sensitive to $T_{\text{core}}$. 

Luminosity

Gravity (mass)

$L \propto M^\alpha$ (empirical)

Hydrostatic Eq.

$T_{\text{surf}} \propto L/R^2$

Thermal Eq.

$T_{\text{core}}$

Pressure (out)

$M \approx R$ if $T_{\text{core}}$ constant

Energy source

Nuclear fusion

Gravitational contraction
Gravity (mass)  
$L \propto M^\alpha$ (empirical)

Luminosity

Hydrostatic Eq.

$T_{\text{surf}} \propto L/R^2$

Wien’s Law

$\lambda_{\text{peak}} \propto T^{-1}_{\text{surf}}$

Pressure (out)

$M \approx R$ if $T_{\text{core}}$ constant

Color

Thermal Eq.

Energy source

Nuclear fusion

Gravitational contraction

Main Sequence Equilibria 9

Gravity

$L \propto M^\alpha$

Luminosity

$T_{\text{surf}} \propto L/R^2$

Hydrostatic Eq.

$T_{\text{core}}$

Pressure (out)

$M \approx R$ if $T_{\text{core}}$ constant

Color

$\lambda_{\text{peak}} \propto T^{-1}_{\text{surf}}$

Wien’s Law

Thermal Eq.

$T_{\text{surf}} \propto L/R^2$

Energy source

Nuclear fusion

Gravitational contraction
Main Sequence Equilibria

Gravity (mass)

Hydrostatic Eq.

Pressure (out)

Energy source

Luminosity

Gravity

Hydrostatic Eq.

Energy source

Luminosity

L \propto M^\alpha

(empirical)

Color

(\lambda_{\text{peak}} \propto T^{-1}\text{surf})

T_{\text{surf}} \propto L/R^2

T_{\text{core}}

T_{\text{surf}}

T_{\text{core}}

T_{\text{core}}

M \approx R

if T_{\text{core}} \text{ constant}

Nuclear fusion

Gravitational contraction

\text{Gravity}

\text{Hydrostatic Eq.}

\text{Pressure (out)}

\text{Energy source}

\text{Nuclear fusion}
How to plot all this information?

Astronomers’ densest plot.
Hertzsprung-Russell Diagram: Axes

(A NTN)
Hertzsprung-Russell Diagram: High-Mass Star?

(ATNF)
Hertzsprung-Russell Diagram: High-Mass Star

(ATNF)
Hertzsprung-Russell Diagram: Low-Mass Star?

(ATNF)
Hertzsprung-Russell Diagram: Low-Mass Star

(ATNF)
Hertzsprung-Russell Diagram: Intermediate-Mass Star(s)?

(ATNF)
Hertzsprung-Russell Diagram

Main Sequence

(Addison Wesley)
Hertzsprung-Russell Diagram

Statistics

(Addison Wesley)
Next: Stellar Evolution

(NAU)